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TECHNICAL NOTE

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A STUDY OF THE EFFECT OF ERRORS IN MEASUREMENT OF VELOCITY AND FLIGHT-PATH ANGLE ON THE GUIDANCE OF A SPACE VEHICLE APPROACHING THE EARTH

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SUMMARY

An analysis was made of the guidance of a space vehicle approaching the earth at supercircular velocity through an entrance corridor containing a desired perigee altitude. Random errors were assumed in the measurement of velocity and flight-path angle and in obtaining the desired thrust impulse. The method described in NASA Technical Note D-191 of scheduling corrections at different values of the angle between perigee and the vehicle's position vector and a slight modification of this method were investigated as a means of correcting perigee altitude when the vehicle's predicted position was at programmed correction points not within a specified deadband about the desired perigee altitude. The study showed that modifying the angular method of NASA Technical Note D-191 by adding another correction near the initial point did not improve the efficiency and accuracy of the angular method.

It was found that in some cases the use of a correction procedure which included a deadband could be more costly in total corrective velocity than a procedure which neglected the deadband. This was especially true if a large degree of confidence was required in the total corrective velocity. It was apparent from the results that a correction with a deadband limit in the guidance scheme was more sensitive to the initial conditions, the corrective procedure, the deadband, and the degree of confidence required than a correction without a deadband limit.

INTRODUCTION

Considerable research has been directed toward the solution of problems associated with guidance to a specified perigee altitude of vehicles approaching the earth at supercircular velocities. The results of some of these studies are presented in references 1 and 2. The basic problem involves the guidance accuracies required in order to place the vehicle in a position to accomplish a successful reentry maneuver. This maneuver might consist of a single-pass entry into the earth's atmosphere without exceeding deceleration and heating limitations or a skipping maneuver in which a parking orbit is attained. To accomplish this maneuver the vehicle must enter the earth's atmosphere within a specified entry corridor (refs. 3 and 4).

In order for a space vehicle approaching the earth at supercircular velocities to intercept the earth's atmosphere at the correct altitude, it may be necessary to apply corrective thrust during midcourse guidance. Reference 1 gives the results of a study of three methods of scheduling corrective-thrust impulses in the presence of assumed random inaccuracies in measuring velocity and flight-path angle and in obtaining the desired thrust impulse. Corrections were applied only at a fixed series of points selected in advance. In reference 1, a correction was applied at each scheduled correction point. The magnitude of each correction was calculated on the basis of the available data on position and velocity to cause the predicted path of the vehicle to reach the desired perigee altitude. In the present study, a similar series of scheduled correction points was employed, but corrections were applied only if the predicted path of the vehicle deviated from the desired perigee altitude by a certain amount, called the deadband. The width of the deadband about the desired perigee altitude decreased as the vehicle approached the earth, as did the assumed measurement errors in velocity and flight-path angle. Comparisons were made of cases with zero deadband, correction to the center of the deadband, and correction to the outer edge of the deadband.

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In the study of reference 2, a constant deadband about the assumed measured value of the trajectory angular rate was employed and corrections were applied at a larger number of positions along the trajectory than in the present study. These differences in formulation prevent a direct comparison of the results of reference 2 with the results of the present study, particularly with regard to the desirability of a deadband.

SYMBOLS

In this paper, distances are measured in U.S. statute miles (1 U.S. statute mile = 1.60935 kilometers).

a	semimajor axis of an ellipse, ft
C	correction point
e	eccentricity of an ellipse
g	gravitational constant, 32.2 ft/sec^2

l	semilatus rectum of an ellipse, ft
R	radius of the earth, 3,960 miles
r	radial distance from center of earth to vehicle, ft, except when used with the standard deviations of the errors in V and γ and then in U.S. statute miles
r_p	radial distance from center of earth to perigee point of flight path, ft
$r_{p,a}$	radial distance from center of earth to perigee point of flight path after final correction, ft
$r_{p,d}$	desired perigee radial distance, ft
V	velocity of space vehicle, ft/sec
V_d	velocity at a given radial distance for the desired trajectory, ft/sec
V_T	magnitude of corrective-velocity vector, ft/sec
$V_{T,i}$	magnitude of corrective velocity that would be required to correct $r_{p,o}$ to $r_{p,d}$ at the initial correction point if $\sigma_V = \sigma_\gamma = \sigma_{V_T} = 0$
α	angle between velocity vector and corrective-velocity vector, deg
ΔV	increment of velocity used to establish deadband, ft/sec
$\Delta\gamma$	increment of flight-path angle used to establish deadband, deg
$\Delta\theta$	change in θ, deg
γ	flight-path angle, deg
γ_d	flight-path angle at a given radial distance for the desired trajectory, deg
θ	angle between a line from center of earth to space vehicle and a line from center of earth to perigee point of flight path, deg
θ_f	magnitude of θ at final correction point, deg

σ	standard deviation of normal distribution
σ_V	standard deviation of error in V, ft/sec
σ_{V_T}	standard deviation of error in V_T, percent
σ_γ	standard deviation of error in γ, deg

Subscripts:

1,2,... 6	order of corrections where 1 is correction at initial position
o	conditions that define initial trajectory

METHOD OF ANALYSIS

Approach Conditions and Assumptions

This study, which was made with a digital computer, is concerned with the portion of an approach trajectory of a space vehicle beginning at $\theta_o = 160^\circ$ and ending at the vacuum perigee radius. In all cases investigated the space vehicle is approaching a planet on an elliptical path with an eccentricity of almost 1.

The space vehicle's characteristics and its reentry procedure will be governing factors in selecting a desired perigee radius and acceptable limits within which the space vehicle must be controlled. In the present study, limits within which the perigee altitude can be controlled for given navigation accuracies are found. No attempt is made to show whether these limits are acceptable or unacceptable for a given vehicle and a given mission; however, these limits are within the entry corridors presented in reference 3.

The following assumptions, the same as those of reference 1, are made in this study:

(1) The earth is spherical.

(2) Motion is considered only in the plane of the orbit for a nonrotating earth.

(3) The space vehicle is close enough to the earth for the gravitation fields of all other bodies to be neglected (a two-body problem).

The basic technique, as illustrated in figure 1, is to schedule observations and apply corrections (if needed) at given angular increments along the approach trajectory to attain the desired perigee altitude. In the analytical study at each observation point the measured values of V and γ (obtained by adding assumed errors to the true values) are used to calculate the orbital characteristics. The calculated perigee radius is compared with a tolerance (the boundaries of the deadband) of the perigee radius to determine if a corrective impulse is needed. If a correction is indicated, calculations are then made to determine the optimum direction and magnitude of corrective velocity required. After adding an assumed error in corrective velocity (error is added only to the magnitude, not direction), the correction is applied in the optimum direction.

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The Monte Carlo technique, described briefly in reference 1 and in more detail in reference 5, was used for the present study. The Taussky-Todd technique for generating a random number on a digitial computer was used to select errors to represent instrumentation inaccuracies in measuring the desired variables.

Equations

The orbital characteristics of a space vehicle approaching the earth on an elliptical path were calculated for the present study with the use of the following equations, which are equations (1) to (5) of reference 1:

$$r = \frac{l}{1 + e \cos \theta}$$

$$V = \sqrt{gR^2\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$\cos \gamma = \sqrt{\frac{l g R^2}{r^2 V^2}}$$

$$e = \sqrt{1 - \frac{l}{a}}$$

$$r_p = a(1 - e)$$

The following expression for the perigee radius in terms of the trajectory variables r, V, and γ (eq. (6) of ref. 1) was used to calculate the deadband within which the space vehicle was to be controlled.

$$r_p = \frac{gR^2\left[1 - \sqrt{1 - \frac{r^2V^2 \cos^2 \gamma\left(2g\frac{R^2}{r} - V^2\right)}{\left(gR^2\right)^2}}\right]}{\frac{2gR^2}{r} - V^2} \tag{1}$$

where $V = V_d + \Delta V$ and $\gamma = \gamma_d - \Delta\gamma$ are used to determine r_p on one side of the deadband and $V = V_d - \Delta V$ and $\gamma = \gamma_d + \Delta\gamma$ are used to determine r_p on the other side.

The present analytical study is based on the application of a thrust impulse in the optimum direction at a given radial distance in order to correct the perigee altitude. When a thrust impulse is applied, the velocity and flight-path angle (dependent upon a known value of r) of the vehicle are changed. This change in V and γ defines a new trajectory with a different perigee altitude. The following analysis is made in order to determine (at any radial distance) the direction in which to apply a corrective thrust that will require the minimum thrust impulse to produce a desired change in perigee altitude.

The total derivative of $r_p(V,\gamma)$ is derived in the appendix of reference 1 and the result is given by the following relation (eq. (7) of ref. 1):

$$dr_p(V,\gamma) = \frac{\partial r_p}{\partial \gamma} d\gamma + \frac{\partial r_p}{\partial V} dV \tag{2}$$

The following relations for dV and $d\gamma$ (obtained from a simple diagram of the velocity vector and the corrective-velocity vector) are true when the error in γ is small and the error in V is small in comparison with V.

$$dV = V_T \cos \alpha \tag{3}$$

$$d\gamma = \frac{V_T \sin \alpha}{V} \tag{4}$$

Substituting these expressions for dV and $d\gamma$ into equation (2) gives the following expression:

$$dr_p = \frac{\partial r_p}{\partial \gamma} \frac{V_T \sin \alpha}{V} + \frac{\partial r_p}{\partial V} V_T \cos \alpha \tag{5}$$

where

$$\frac{\partial r_p}{\partial \gamma} = - \frac{l}{e} \tan \gamma$$

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and

$$\frac{\partial r_p}{\partial V} = \frac{2gR^2(1 - e)}{e\left(2g\frac{R^2}{r} - V^2\right)^2}\left[\frac{g\frac{R^2}{r}(1 + e)}{V} - V\right]$$

The maximum change in r_p is obtained when the corrective velocity is made in the direction defined by

$$\tan \alpha = \frac{\frac{\partial r_p}{\partial \gamma}}{\frac{\partial r_p}{\partial V} V} \tag{6}$$

From equations (2), (5), and (6), a minimum value of the magnitude of the corrective-velocity vector may be found to produce a given change in the perigee altitude.

When relatively large changes in V and γ are required to correct the error in r_p, the relations for dV and $d\gamma$ (eqs. (3) and (4)) introduce errors in calculating V_T. Therefore, in order to minimize the effect of this error, the following iteration procedure was used to calculate the corrective-velocity vector.

The corrective-velocity vector was calculated from the preceding equations and this correction was assumed to be made. The values of r_p and other variables of the trajectory that the vehicle would obtain with this correction were determined. If the error in r_p was greater than 10,000 feet, the calculated corrective velocity given by a second application of these equations was added to this correction by using the trajectory variables calculated in the previous step and the perigee altitude was recalculated. This process was continued until the error in r_p was less than or equal to 10,000 feet. The summation of these corrective-velocity vectors was then used as the corrective velocity applied at the local correction point.

Range of Initial Variables

The errors in measuring velocity and flight-path angle and the error in applying corrective thrust are assumed to have a normal distribution. The two sets of assumed errors of this report were selected from the errors investigated in reference 1. The errors that were dependent upon range were selected because available information indicated that the accuracy of measuring the velocity vector of a vehicle approaching the earth would be a function of the distance the vehicle is from the earth. The standard deviations of the errors investigated were:

First	Second
$\sigma_V = \frac{r}{10,000}$ ft/sec	$\sigma_V = \frac{3r}{10,000}$ ft/sec
$\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees	$\sigma_\gamma = \frac{0.0375r}{10,000}$ degrees
$\sigma_{V_T} = 1.3$ percent	$\sigma_{V_T} = 3.9$ percent

Initial conditions were assumed so that without corrections perigee radii of 0.75R, 0.99R, 1.01R, 1.25R, 1.5R, and 2R would be obtained.

The three deadband widths studied for each set of instrumentation inaccuracies were those obtained when (1) $\Delta V = \sigma_V$ and $\Delta\gamma = \sigma_\gamma$, (2) $\Delta V = 3\sigma_V$ and $\Delta\gamma = 3\sigma_\gamma$, and (3) $\Delta V = 0$ and $\Delta\gamma = 0$ (zero deadband). Throughout this report these deadbands, which were dependent upon range, are referred to as (1) σ deadband, (2) 3σ deadband, and (3) zero deadband.

Method of Control

Two methods of perigee control, the angular method of reference 1 and a modification of this method (see fig. 1), were investigated. The first method of scheduling observation points was to make an observation and apply a correction if needed each time the true anomaly increased a given amount. The angles used were $\theta_o = 160^\circ$ and $\Delta\theta = 30^\circ$. The results of reference 1 indicated that a modification of the angular method such that an additional correction is applied close to the initial point may lower the total-corrective-velocity requirements when large errors are assumed in the initial perigee altitude. Therefore, the second method of the present study, which is referred to as the modified angular method, was the same as the angular method except another observation point was added relatively close to the initial point. For this method, the angular increment was 5° between the initial observation point and the second observation point, 25° between the second and third observation points, and 30° between the other points.

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The present study was made with the use of constant angular increments to schedule correctional maneuvers for perigee-altitude control. (This procedure is called the angular method.) After the digital computer program used for the study reported in reference 1 had been modified for the present study, the author's attention was called to reference 6 in which the effects upon correctional maneuvers of errors in the predicted trajectory, the observed position and velocity of a space vehicle, and in the application of velocity corrections are considered and the optimal frequency of making corrections are determined. The conclusion of reference 6 is that a correctional maneuver program will be near optimal if correctional maneuvers are carried out at times which, together with the first maneuver, are separated from the time of arrival by intervals in a geometric progression, the common ratio of which is about 1:3.

Before the present study was continued, calculations were made to determine the ratios of the time intervals between the correction points used in the angular method. The ratio of the time intervals, when $\Delta\theta = 30^\circ$ was used, varied from 1:4.3 to 1:2.1, with an average ratio of 1:2.9.

The other correction procedures used in reference 1 did not approximate the ratio of time intervals of 1:3. However, since the favorable results obtained with the angular method tend to agree with the theoretical conclusions of reference 6, the present study was continued with the same method of correction.

In order to determine the corrective velocity to be applied, it was assumed that the vehicle was on the trajectory defined by the measured values (true value plus a random error) of V and γ. Calculations were then made to determine the magnitude of corrective velocity needed to correct the measured trajectory. The two corrective procedures for both methods of space-vehicle control were the application of a velocity to correct the vehicle from the measured trajectory to the center of the deadband and the application of a velocity to correct the vehicle to the nearest boundary of the deadband. It should be noted that when the vehicle is assumed to be on the trajectory defined by the measured values of V and γ and an attempt is made to correct the vehicle to the center of the deadband, it is actually being corrected to a trajectory that is in error in relation to the desired trajectory. Likewise, when an attempt is made to correct the vehicle to the nearest boundary of the deadband, it is being corrected to a trajectory that is in error in relation to the desired trajectory (center of the deadband) by one-half the deadband width plus the error resulting in the errors incurred in measuring V and γ. Thus, if there were no errors in applying the corrective velocity, the probability of overcorrecting the desired trajectory would be greater when attempting to correct to the center of the deadband than when attempting to correct to the nearest boundary.

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RESULTS AND DISCUSSION

General Discussion

The results of this study, presented in figures 2 to 13, are shown as probability-distribution curves. Since the equations of motion and the equations for the change in perigee radius are not dependent upon a specific value of perigee altitude, the probability that the perigee altitude is less than the desired perigee altitude plus or minus a given value can be read directly from the perigee-altitude probability curves. The probability that the total corrective velocity is less than a given value of V_T can be read directly from the probability-distribution curves of total corrective velocity. The values of V_T presented in this report are based on a desired perigee altitude of 250,000 feet.

All the perigee-altitude results of this report are based on 1,000 runs. The procedure, which was programmed on the digital computer, was used as an aid for determining the statistical distribution of the results of a large number of perigee-altitude runs but it did not prove to be satisfactory in the case of the total-corrective-thrust results. Hence, a comparison between total-corrective-velocity probability curves based on 100 runs and probability curves based on 1,000 runs was made. This comparison, shown in figure 2 for four cases where $\sigma_V = \frac{3r}{10{,}000}$ ft/sec, $\sigma_\gamma = \frac{0.0375r}{10{,}000}$ degrees, $\sigma_{V_\Gamma} = 3.9$ percent, $\Delta V = \sigma_V$, $\Delta\gamma = \sigma_\gamma$, $r_{p,o} = 2.0R$, and the final correction point was at $\theta_f = 10^\circ$, shows good agreement between the curves based on 100 runs and those based on 1,000 runs. Thus, the remainder of the total-corrective-velocity probability curves of this study are based upon 100 runs, a value small enough to be analyzed manually.

When the total-corrective-velocity data were plotted, a continuous curve was faired through the data except when several runs required the same or approximately the same value of V_T and this was shown by a step or an abrupt change in slope of the probability curve. Attention is called to the step from zero probability to a given percent probability shown in the total-corrective-velocity probability-distribution curves. This step simply means that a certain percent of the runs required only one correction. For example, figure 2(a) shows that approximately 7 or 8 percent of the runs required one correction of 506 ft/sec in order to accomplish the mission.

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Comparison of Two Methods of Control

A series of cases where both methods of control were used were analyzed in order to determine the effect of modifying the angular method of control. Figure 3 gives the results of cases where $r_{p,o} = 1.25R$, the errors in V, γ, and V_T were $\sigma_V = \frac{r}{10,000}$ ft/sec, $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees, and $\sigma_{V_T} = 1.3$ percent, and the final correction point was at $\theta_f = 10^\circ$. The results show that the total-corrective-velocity requirements of both methods of control are about the same, and both methods of control had the same perigee-altitude error distribution. This was found to be true for all initial conditions studied; therefore, results for the modified angular method will not be included in the remainder of the report.

Effect of the Deadband Corrective Procedure

A series of cases were analyzed in order to determine the effect of making corrections to the center of the deadband and to the nearest boundary of the deadband. These results, giving the perigee-altitude error in figure 4 and total corrective velocity in figure 5, show a perigee-altitude band of approximately the same width for both types of corrections. However, when an attempt is made to correct the vehicle to the nearest boundary, the perigee-altitude distribution shifts from a mean value of almost zero to a mean value that is positive if the initial value of $r_{p,o}$ is too large and negative if the initial value of $r_{p,o}$ is too small. Figure 5 shows that the total corrective velocity required for 100-percent probability was lower for cases where corrections were made to the nearest boundary of the deadband than for cases where corrections were made to the center of the deadband.

Cases were analyzed for three deadband widths in order to determine the effect of the width of the deadband on the distribution of perigee altitude and total corrective velocity. The widths of the deadband were based upon the assumed measurement errors in velocity and flight-path angle. The three widths studied were selected in such a manner that the values of ΔV and $\Delta\gamma$ in equation (1) were equal to either the standard deviation in V and γ, three times the standard deviation in V and γ, or zero (no deadband). The results of cases where three L
widths of the deadband were used for the angular method of scheduling 1
observations are shown as perigee-altitude-error distribution curves in 6
figure 6 and as total-corrective-velocity distribution curves in fig- 6
ure 7. These results indicate that as the width of the deadband increases, 1
the width of the perigee-altitude band and the corrective-velocity requirements both increase markedly. Thus, it was concluded that a deadband that is obtained by using three times the standard deviation in V and γ would not be satisfactory.

The distribution curves for perigee-altitude error and total corrective velocity, presented as figures 8 and 9, respectively, show the results of cases where $r_{p,o}$ = 0.99R, 1.01R, 1.5R, and 2.0R for the σ deadband and the zero deadband. These results show that when the zero deadband rather than the σ deadband was used, a smaller perigee-altitude band is obtained for slightly lower or approximately the same total-corrective-velocity requirements.

In order to study the effect of the deadband width further, a series of cases were run in which the final observation point of the angular method was omitted. Results of cases which have the same instrumentation error, initial perigee altitudes, and correction to the center of the deadband for the angular method making the final observation at $\theta = 40^\circ$ are shown in figures 10 and 11. These results also indicate that a smaller perigee-altitude band (see fig. 10) at lower or approximately the same total-corrective-velocity requirements (see fig. 11) is obtained for the zero deadband than for the σ deadband.

A comparison of the results of figures 10 and 11 with results of figures 6 to 9 shows the effect of the location of the final correction point. It is of interest to note that this comparison shows that a perigee-altitude band of about the same width (compare figs. 6 and 8 with fig. 10) is obtained at a lower total corrective velocity (compare figs. 7 and 9 with fig. 11) for the zero deadband where $\theta_f = 40^\circ$ and for the σ deadband where $\theta_f = 10^\circ$.

Effect of Instrumentation Errors

A series of cases, where the σ deadband and zero deadband were used, were analyzed in order to determine the effects on the perigee altitude and total corrective velocity of increasing the assumed instrumentation errors. Figures 12 and 13 give the results of cases where the errors were:

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First	Second
$\sigma_V = \dfrac{r}{10{,}000}$ ft/sec	$\sigma_V = \dfrac{3r}{10{,}000}$ ft/sec
$\sigma_\gamma = \dfrac{0.0125r}{10{,}000}$ degrees	$\sigma_\gamma = \dfrac{0.0375r}{10{,}000}$ degrees
$\sigma_{V_T} = 1.3$ percent	$\sigma_{V_T} = 3.9$ percent

These results indicate that by multiplying the instrumentation errors by 3, the perigee-altitude band tripled in width and the total corrective velocity required for 100-percent probability more than tripled.

Explanation of Results

An explanation of the results was found by examining corrections made at each observation point along the approach trajectory. For the case where a zero deadband and a large initial error in perigee altitude were assumed, a large correction was made at the initial observation point. At each additional observation point, a correction was made which was small in comparison with the first correction so that the total corrective velocity was slightly larger than the initial correction.

For similar cases where a σ deadband was used, the initial correction was similar in magnitude to the case where a zero deadband was used. The vehicle would then proceed along its trajectory with a perigee-altitude error that was hidden by instrumentation errors and the deadband, and no additional corrections would be made until this error was detected. If this error was small enough so that it was not detected at the final observation point, the total corrective velocity required for that run would be less for the σ deadband than for a zero deadband. For example, figure 7 shows that the probability of the σ deadband with less corrective velocity than the zero deadband was about 30 percent when $r_{p,o} = 0.75R$ and about 75 percent when $r_{p,o} = 1.25R$. When the perigee-altitude error after initial correction is large enough to be detected at an observation point close to the final perigee altitude, a large correction is required to correct to the desired perigee altitude. These final corrections when $\theta_f = 10^\circ$ and 95- to 100-percent probability is desired were always large enough to make the total corrective velocity greater for the σ deadband than for the case where a zero deadband was used.

For small initial-perigee-altitude errors and for a σ deadband, no correction was generally needed at the first few observation points and later corrections were larger than those needed with a zero deadband.

SUMMARY OF RESULTS

A study of the effects of errors in velocity and flight-path angle on the guidance of a space vehicle approaching the earth was made. The vehicle's predicted position with respect to a deadband about a desired perigee altitude determined if a corrective maneuver was made at each of the scheduled correction points along the trajectory. This study indicates the following results:

1. By using a correction procedure which omitted the deadband, the total corrective velocity required for a high-percent probability of not using more than a given total corrective velocity was less for most cases than a procedure which included a deadband. Better perigee-altitude control was achieved when the deadband was omitted.

2. A correction with a deadband limit was more sensitive to the initial conditions, instrumentation inaccuracies, location of final correction point, and degree of confidence required than a correction without a deadband limit.

3. These results indicated that when a deadband was used, it would be more efficient to correct to the nearest boundary of the deadband than to the center of the deadband if the shift in perigee altitude can be tolerated.

4. The addition of another correction point near the initial point of this study did not improve the efficiency and accuracy of the angular method.

5. A deadband that was based on 3 times the standard deviations of the errors in space-vehicle velocity and flight-path angle would not be satisfactory because of the high corrective-thrust requirement and poor perigee-altitude control.

6. These results indicated that by tripling the instrumentation errors, the perigee-altitude band tripled in width and the total corrective velocity required more than tripled.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Air Force Base, Va., July 11, 1961.

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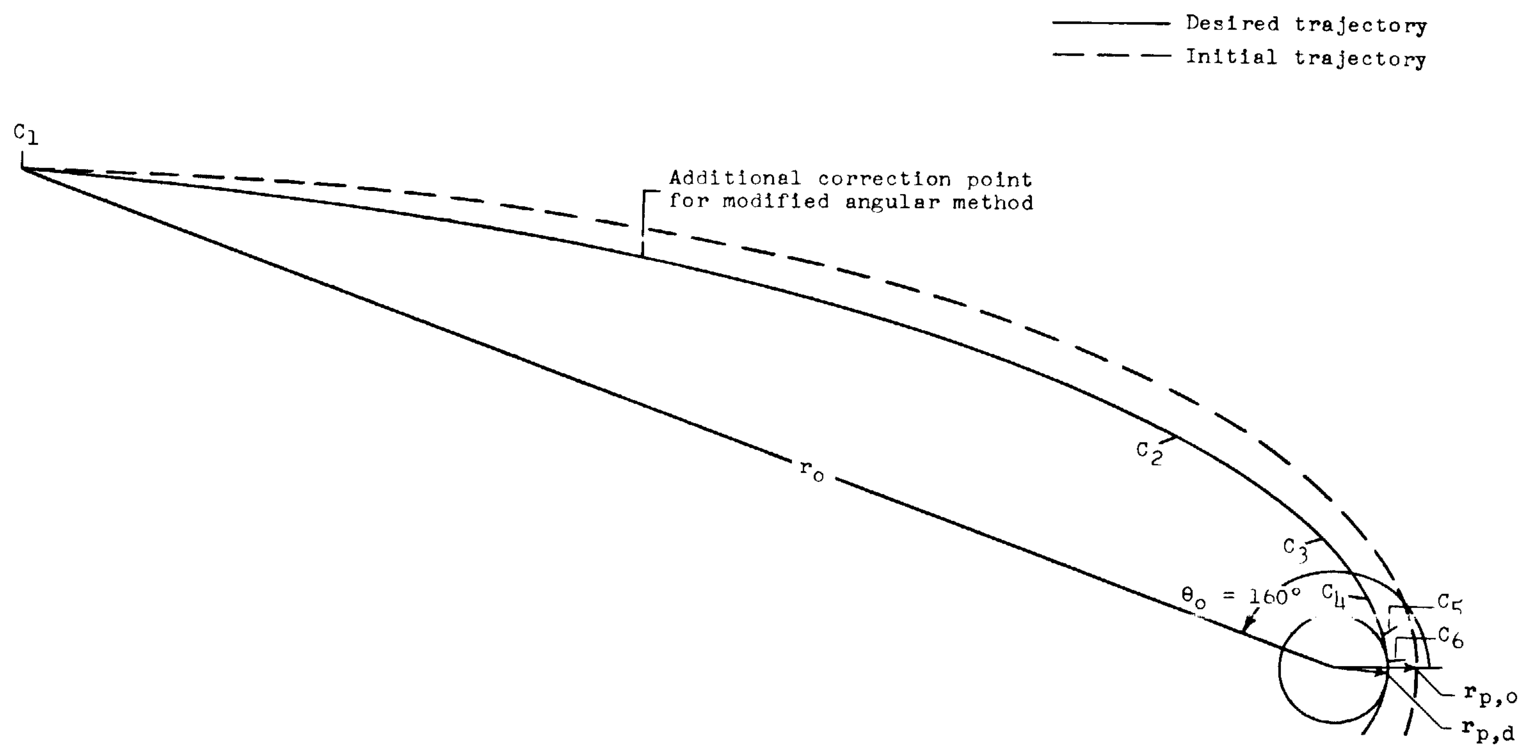

Figure 1.- Diagram which shows the correction points along the desired trajectory and the initial trajectory where $r_{p,o} = 1.5R$.

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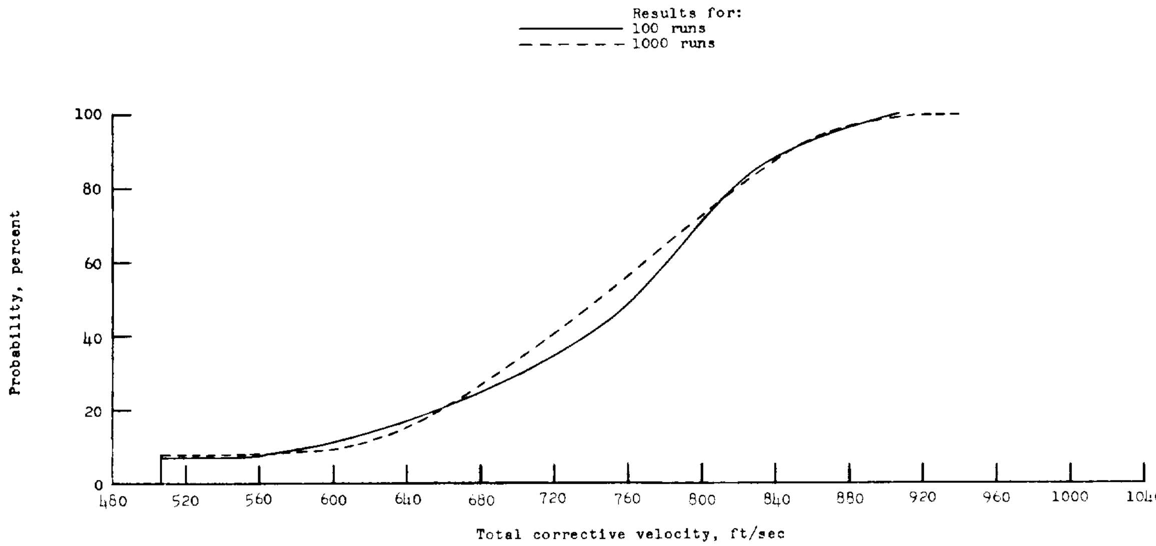
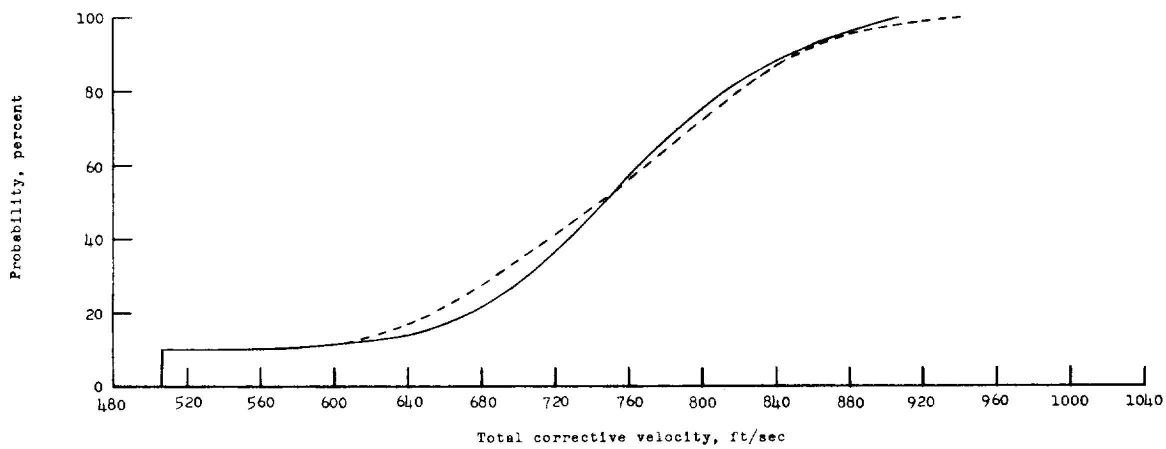
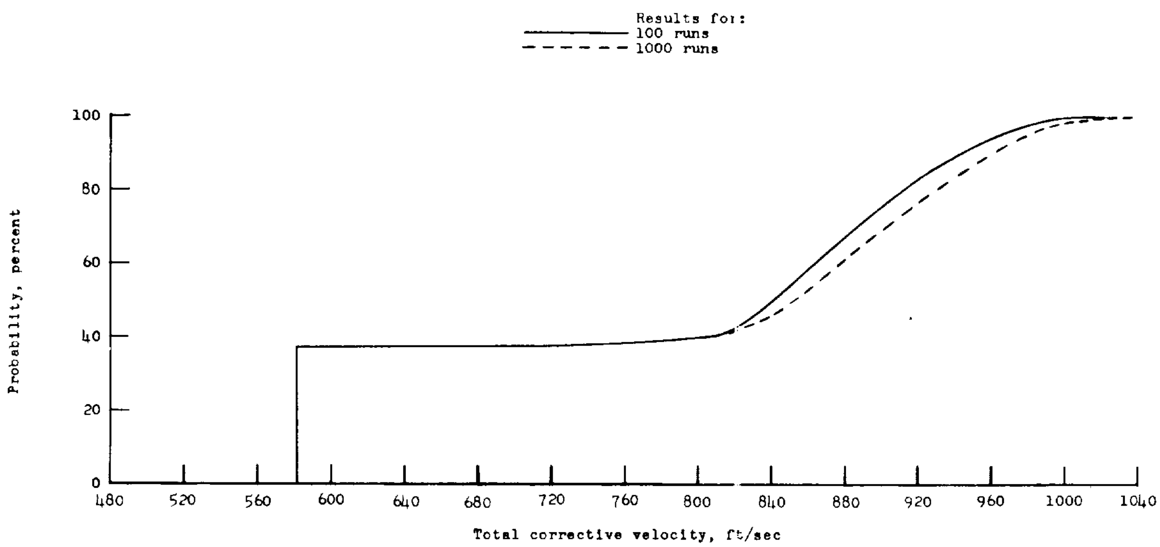


(a) Corrections made to center of deadband with angular method.

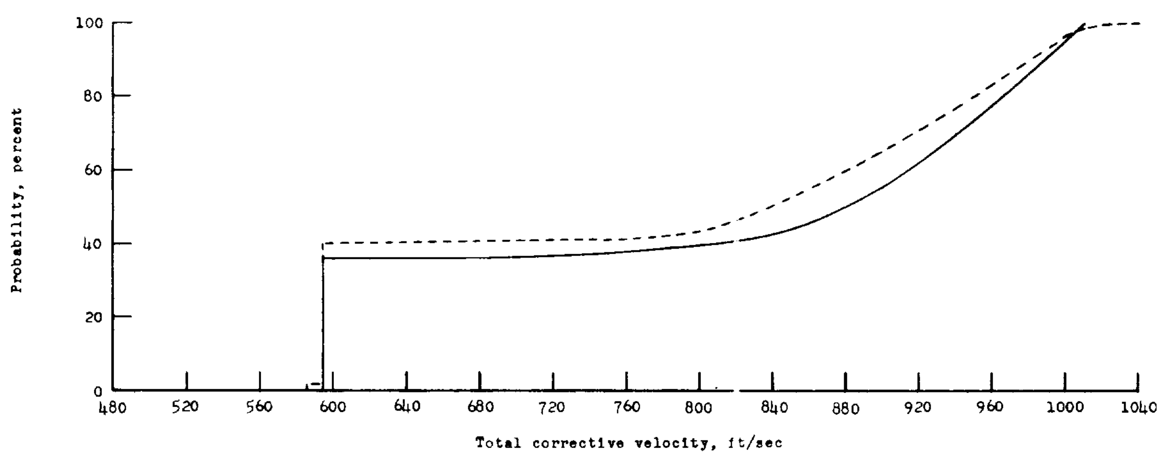


(b) Corrections made to center of deadband with modified angular method.

Figure 2.- Total-corrective-velocity probability curves based on 100 runs and 1,000 runs for the angular method and modified angular method when corrections were made to the center of the deadband and to the nearest boundary of the deadband. $\sigma_V = \frac{3r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0375r}{10,000}$ degrees; $\sigma_{V_T} = 3.9$ percent; $r_{p,o} = 2.0R$; σ deadband; $\theta_f = 10^\circ$; $V_{T,i} = 510$ ft/sec.



(c) Corrections made to boundary of deadband with angular method.



(d) Corrections made to boundary of deadband with modified angular method.

Figure 2.- Concluded.

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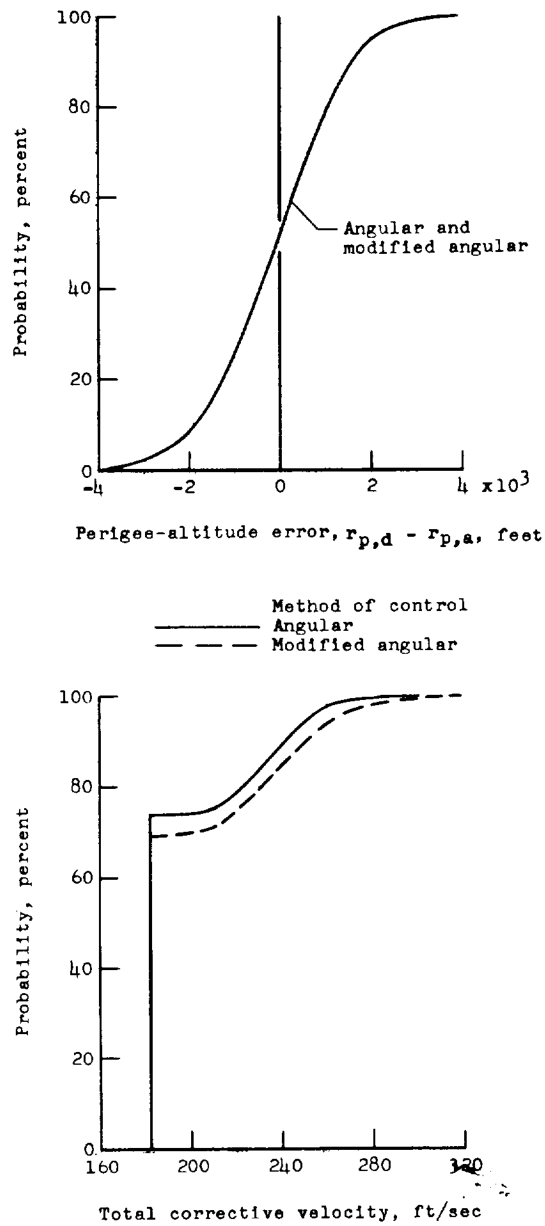
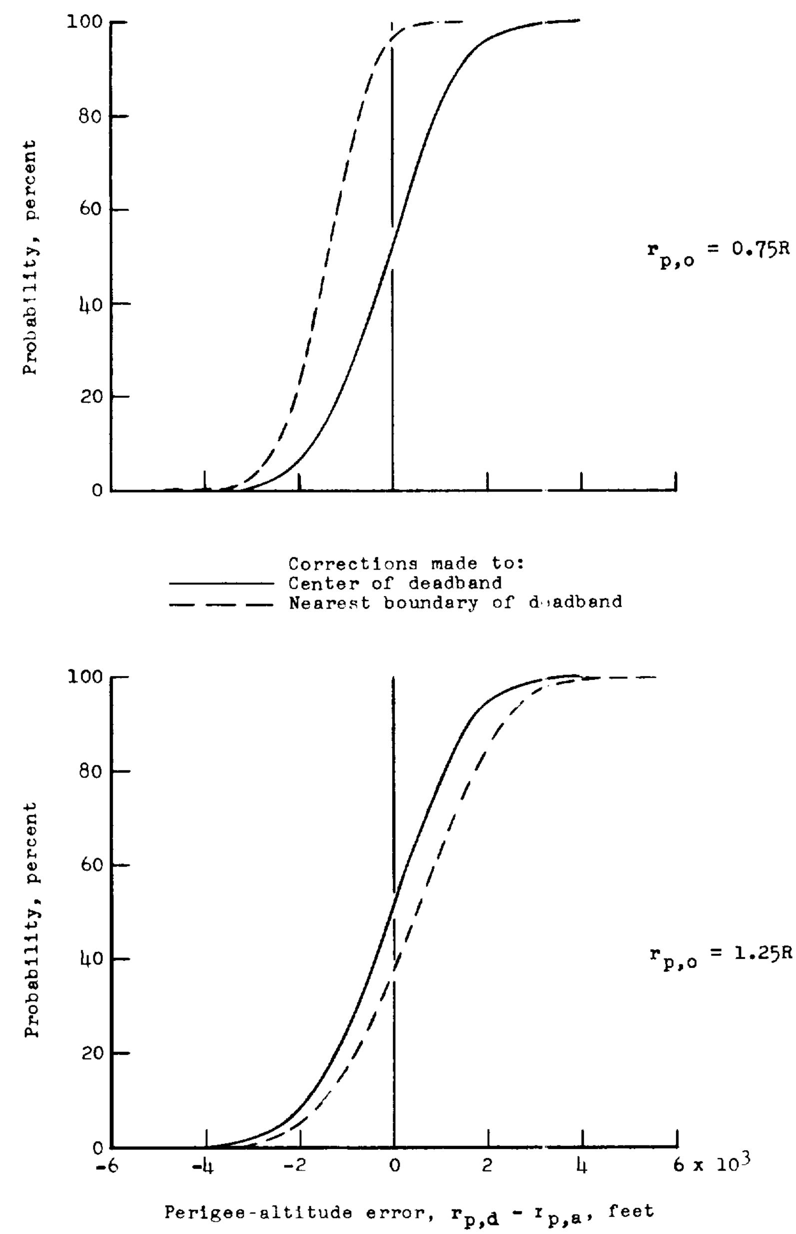


Figure 3.- Probability-distribution curves for the angular and modified angular methods of perigee-altitude control where corrections were made to the center of deadband. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 1.3$ percent; σ deadband; $r_{p,o} = 1.25R$; $\theta_f = 10^{\circ}$; $V_{T,i} = 182$ ft/sec.



(a) $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 1.3$ percent.

Figure 4.- Probability-distribution curves of perigee-altitude error for the angular method when corrections were made first to the center of the deadband and second to the nearest boundary of the deadband. σ deadband; $\theta_f = 10^\circ$.

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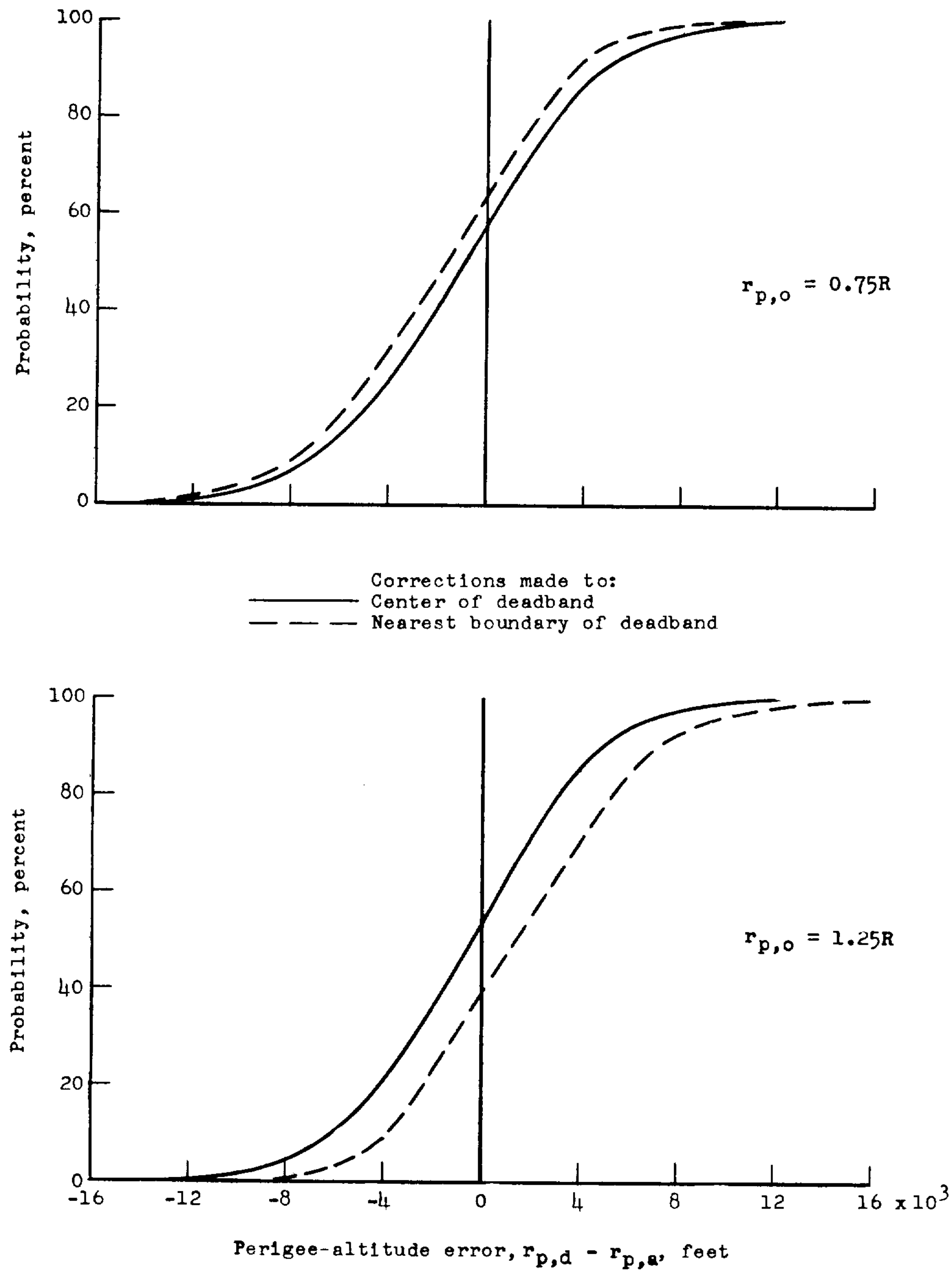
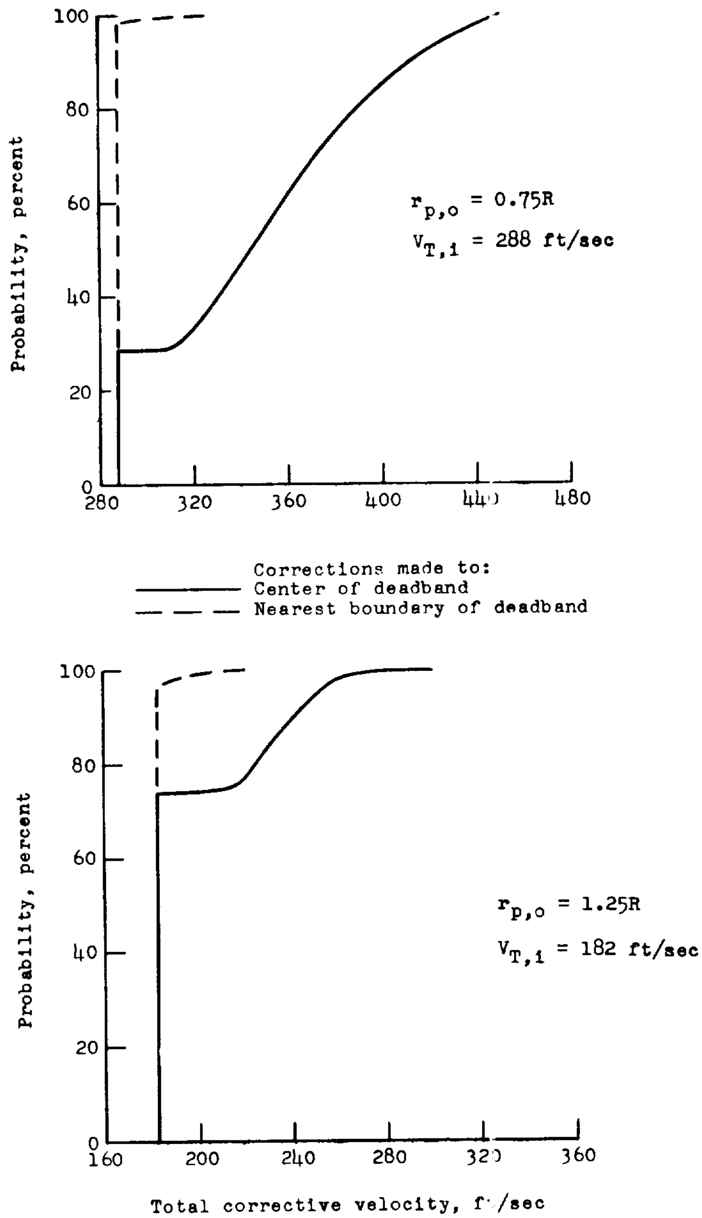


(b) $\sigma_V = \dfrac{3r}{10{,}000}$ ft/sec; $\sigma_\gamma = \dfrac{0.0375r}{10{,}000}$ degrees; $\sigma_{V_T} = 3.9$ percent.

Figure 4.- Concluded.



(a) $\sigma_V = \dfrac{r}{10,000}$ ft/sec; $\sigma_\gamma = \dfrac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 1.3$ percent.

Figure 5.- Total-corrective-velocity probability-distribution curves for the angular method when corrections were made first to the center of the deadband and second to the nearest boundary of the deadband. σ deadband; $\theta_f = 10^{\circ}$.

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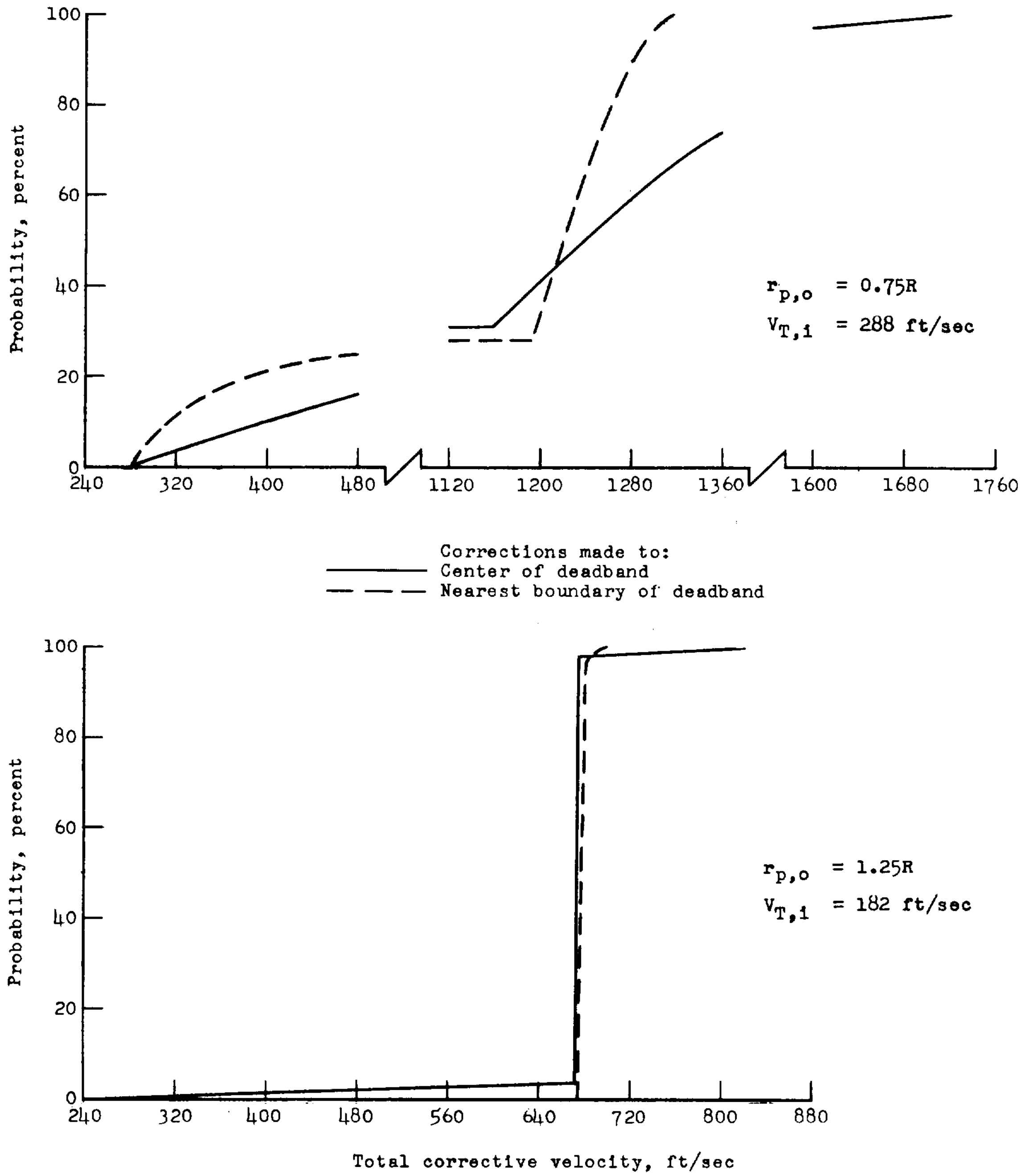

(b) $\sigma_V = \frac{3r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0375r}{10,000}$ degrees; $\sigma_{V_T} = 3.9$ percent.

Figure 5.- Concluded.

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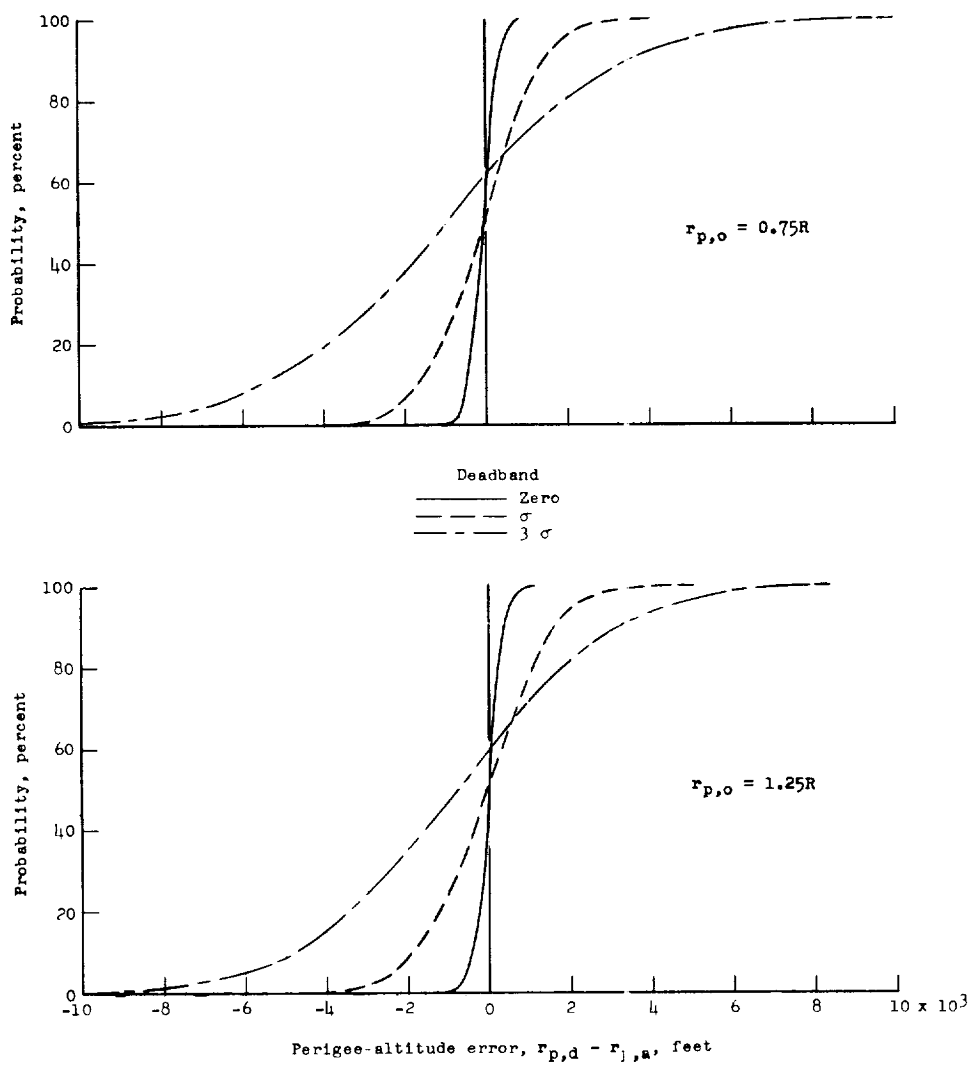

Figure 6.- Probability-distribution curves of perigee-altitude error for three deadbands using the angular method where corrections were made to the center of deadband. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 1.3$ percent; $\theta_f = 10^\circ$.

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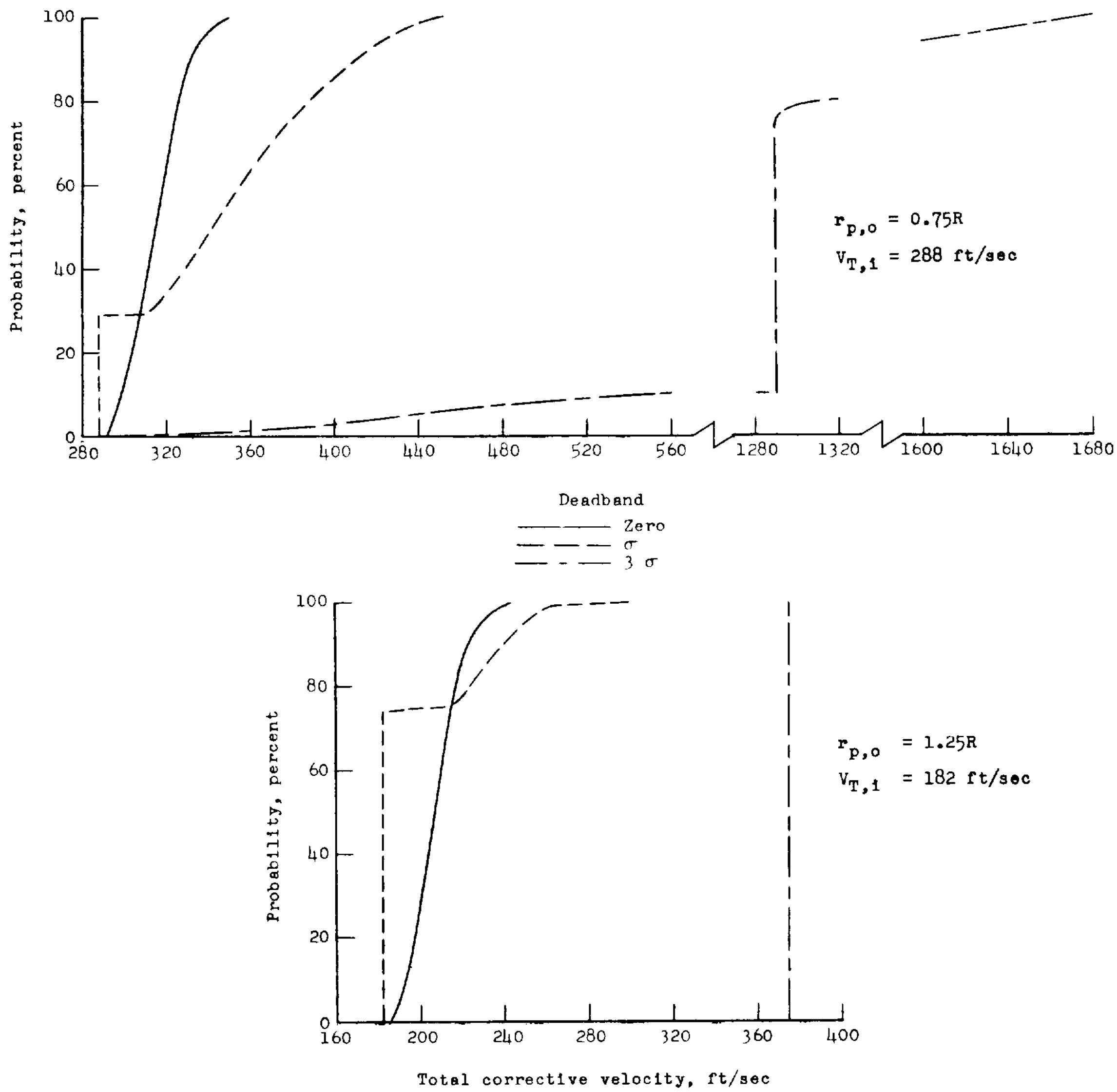

Figure 7.- Total-corrective-velocity probability-distribution curves for three deadbands using the angular method where corrections were made to the center of the deadband. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; σ_{V_T} = 1.3 percent; $\theta_f = 10^\circ$.

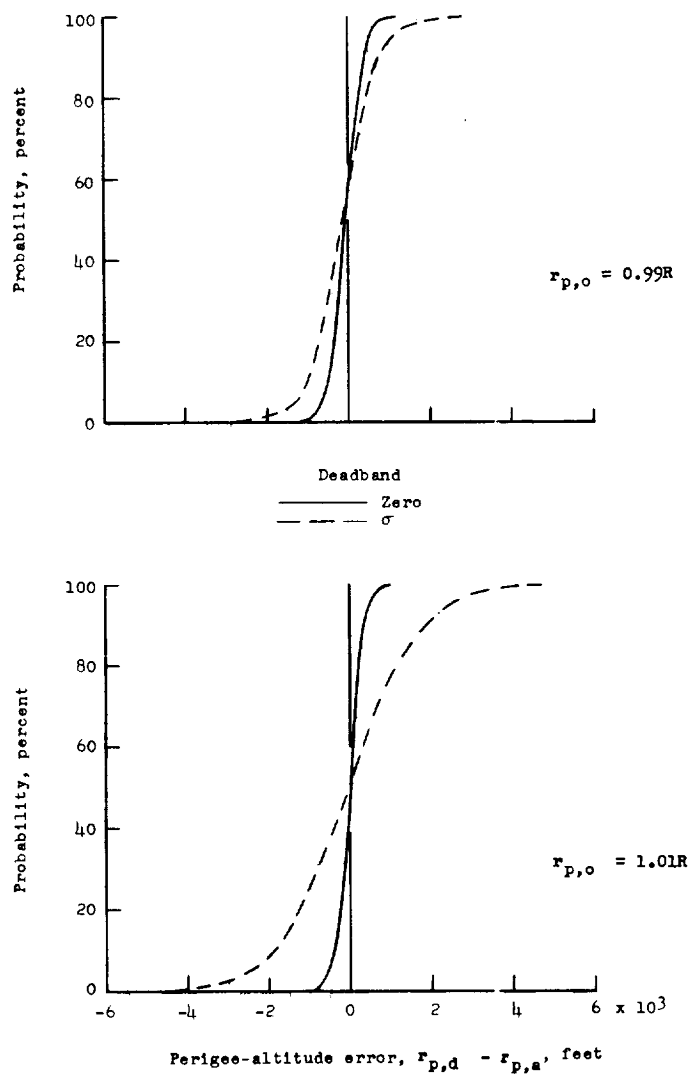

Figure 8.- Probability-distribution curves of perigee-altitude error for σ and zero deadbands with the angular method where corrections were made to the center of the deadband. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 1.3$ percent; $\vartheta_f = 10^\circ$.

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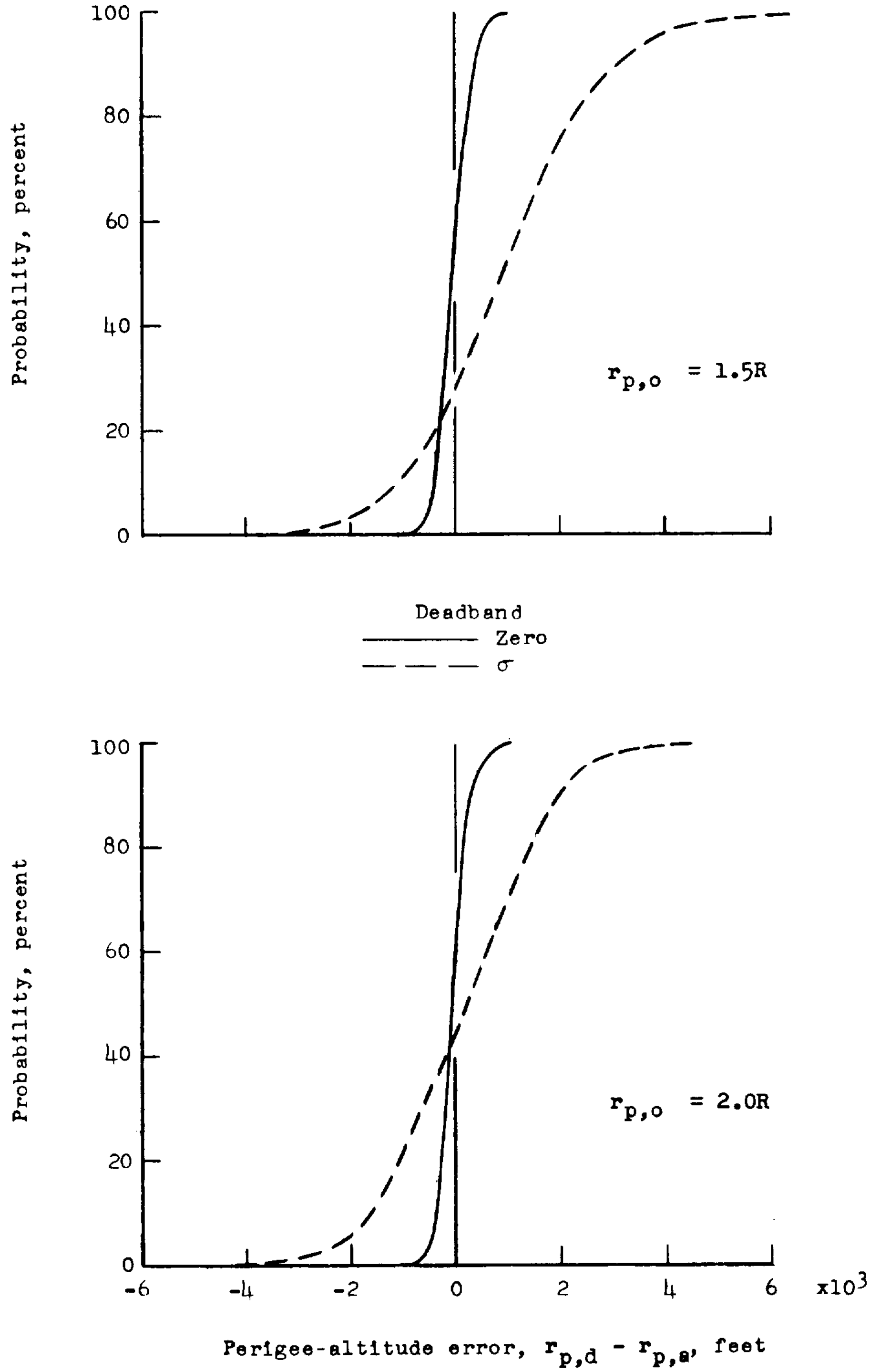

Figure 8.- Concluded.

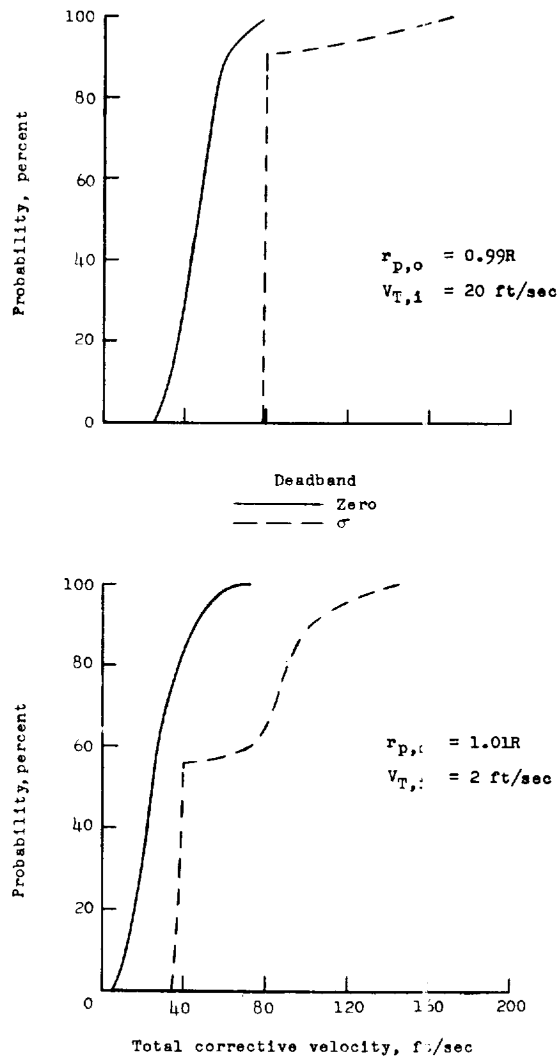

Figure 9.- Total-corrective-velocity probability-distribution curves for σ and zero deadbands using the angular method where corrections were made to the center of the deadband. $\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 1.3$ percent; $\theta_f = 10^\circ$.

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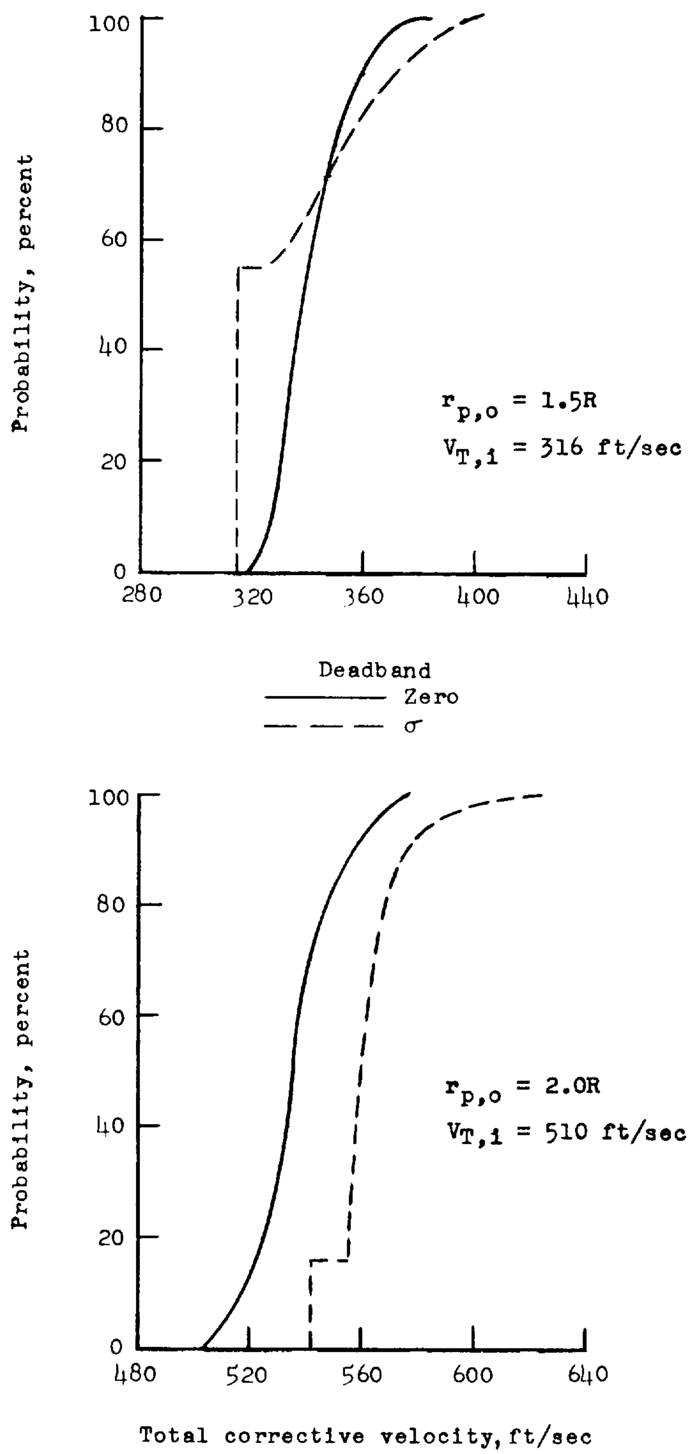

Figure 9.- Concluded.

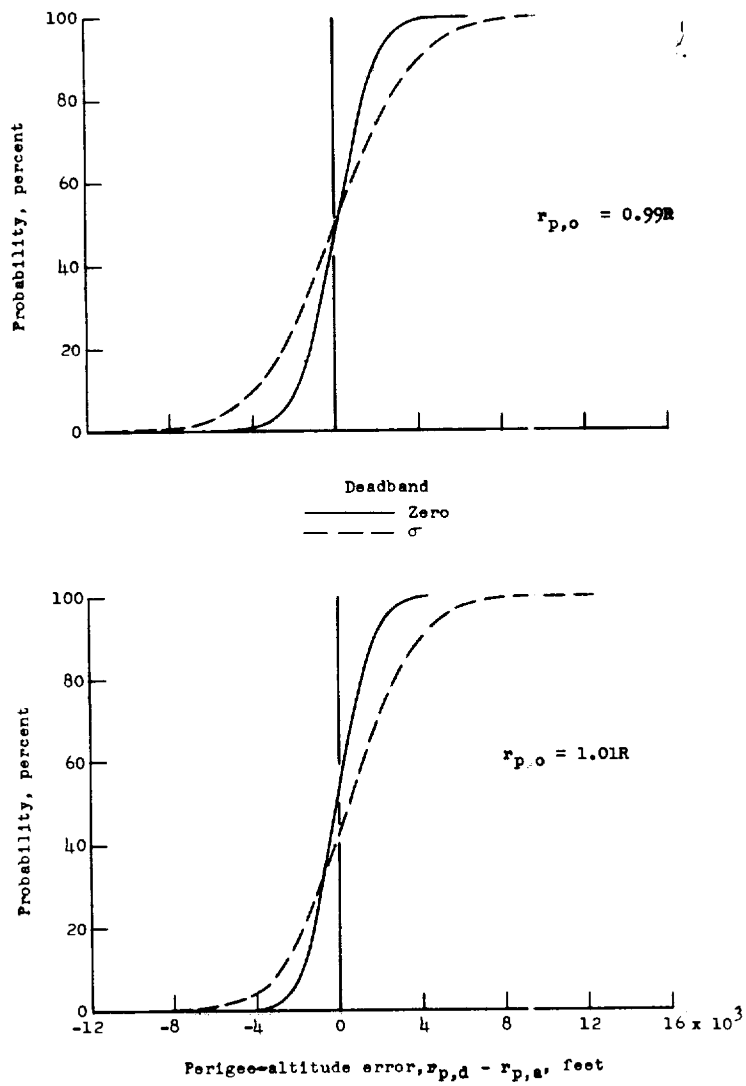

Figure 10.- Probability-distribution curves of perigee-altitude error for σ and zero deadbands with $\theta_f = 40^\circ$ for the angular method where corrections were made to the center of the deadband.

$\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 1.3$ percent.

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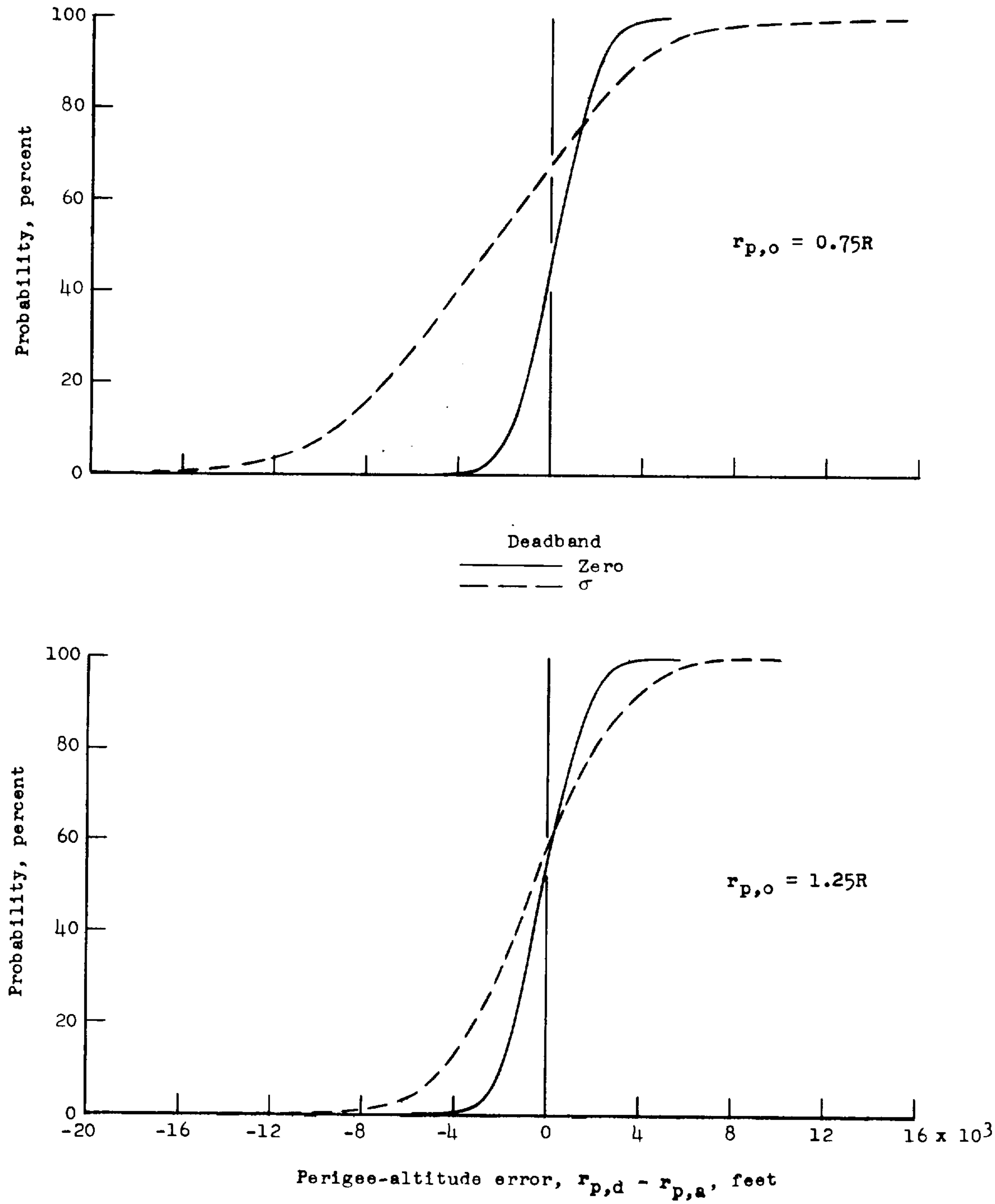

Figure 10.- Continued.

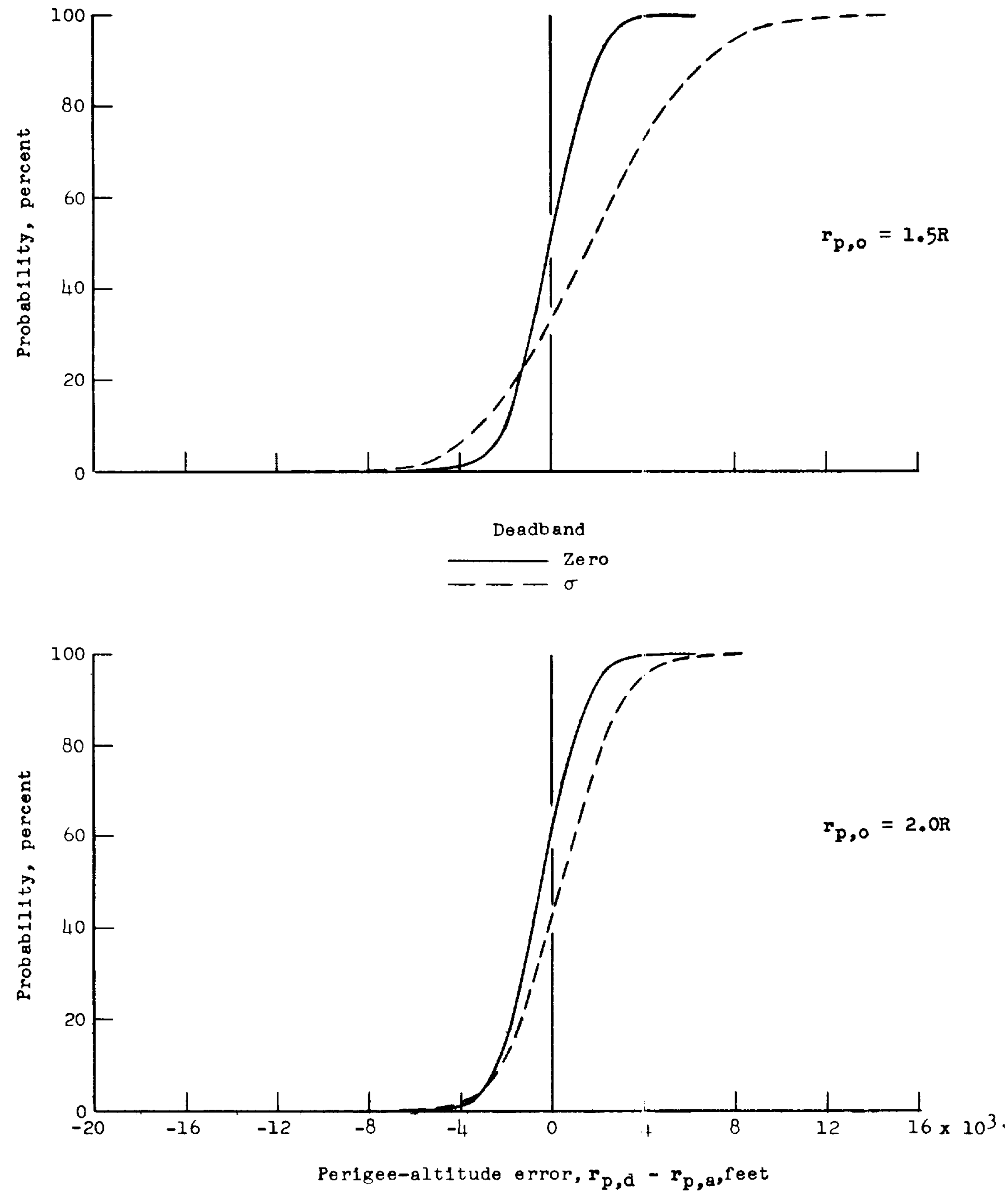

Figure 10.- Concluded.

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3D

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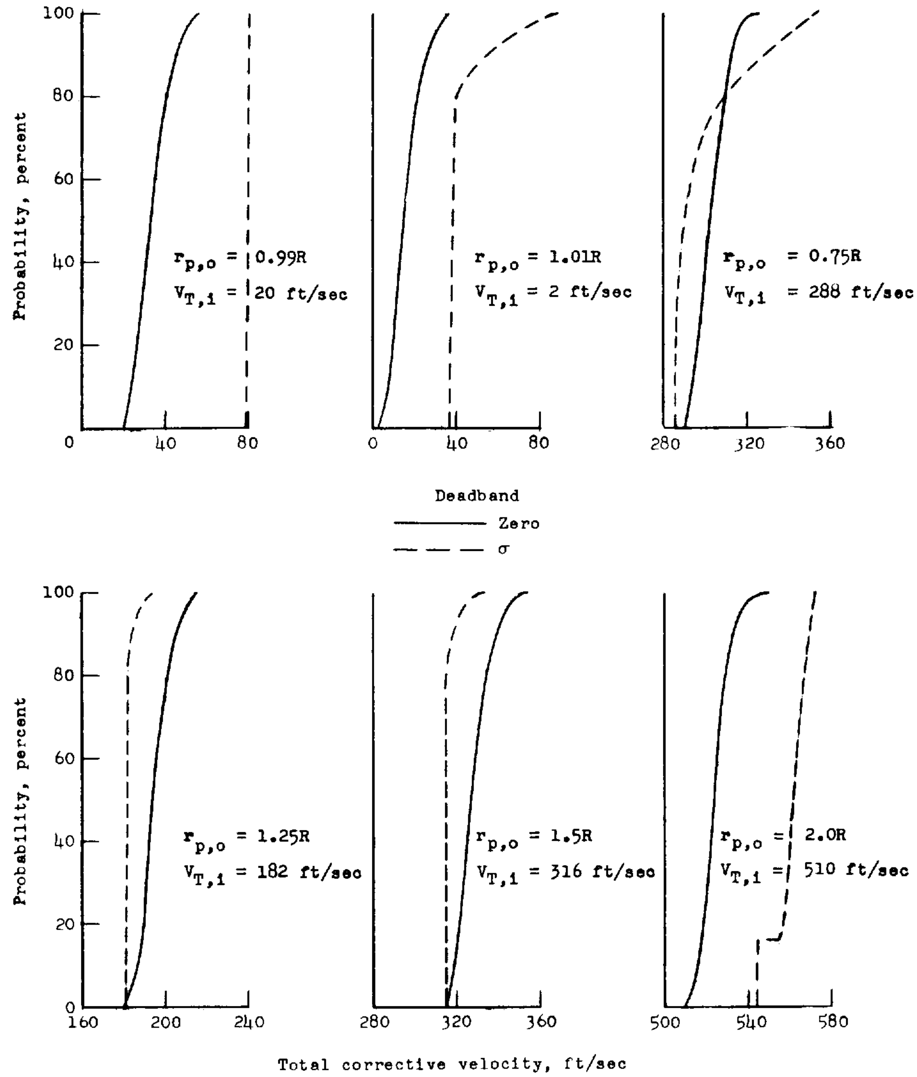
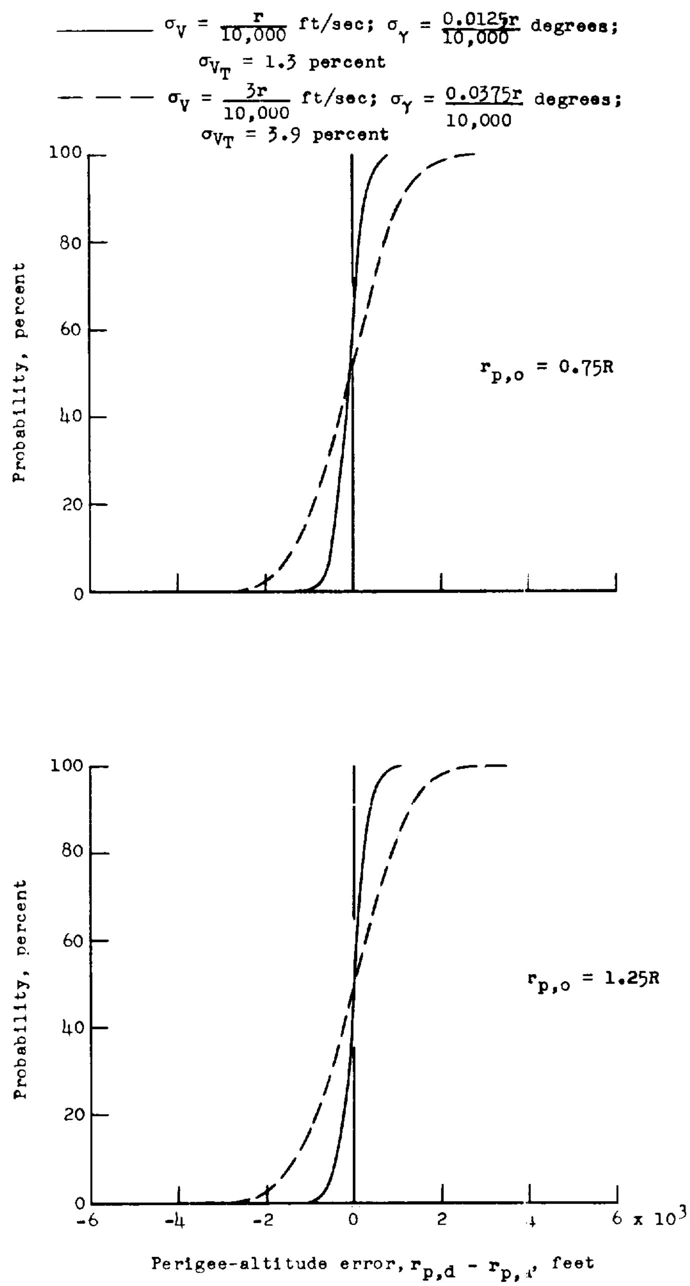


Figure 11.- Total-corrective-velocity probability-distribution curves for σ and zero deadbands with $\theta_f = 40^\circ$ for the angular method where corrections were made to the center of the deadband.

$\sigma_V = \frac{r}{10,000}$ ft/sec; $\sigma_\gamma = \frac{0.0125r}{10,000}$ degrees; $\sigma_{V_T} = 1.3$ percent.



(a) Zero deadband.

Figure 12.- Probability-distribution curves of perigee-altitude error for σ and zero deadbands with the angular method where corrections were made to the center of the deadband for two sets of instrumentation inaccuracies. $\theta_f = 10^\circ$.

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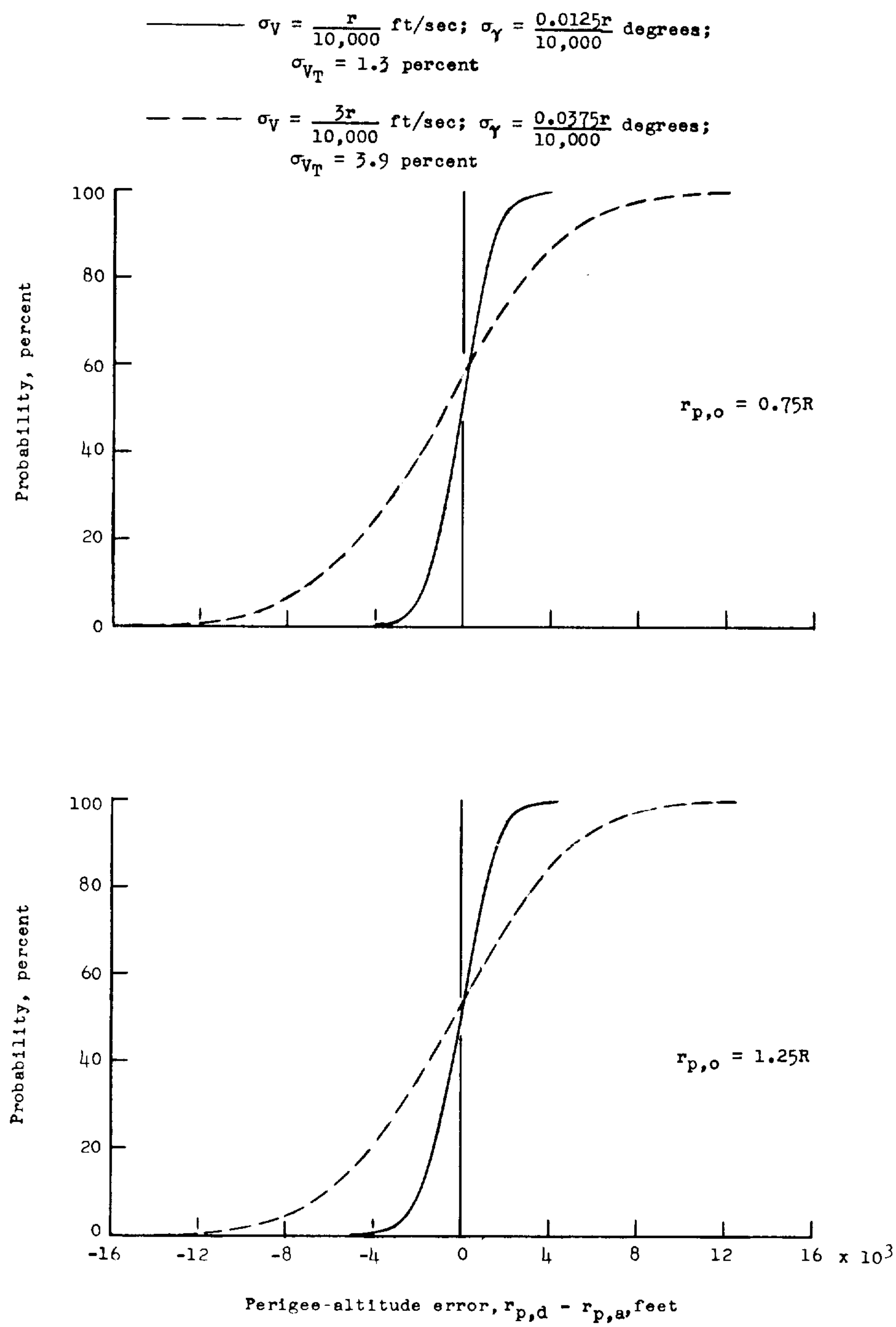
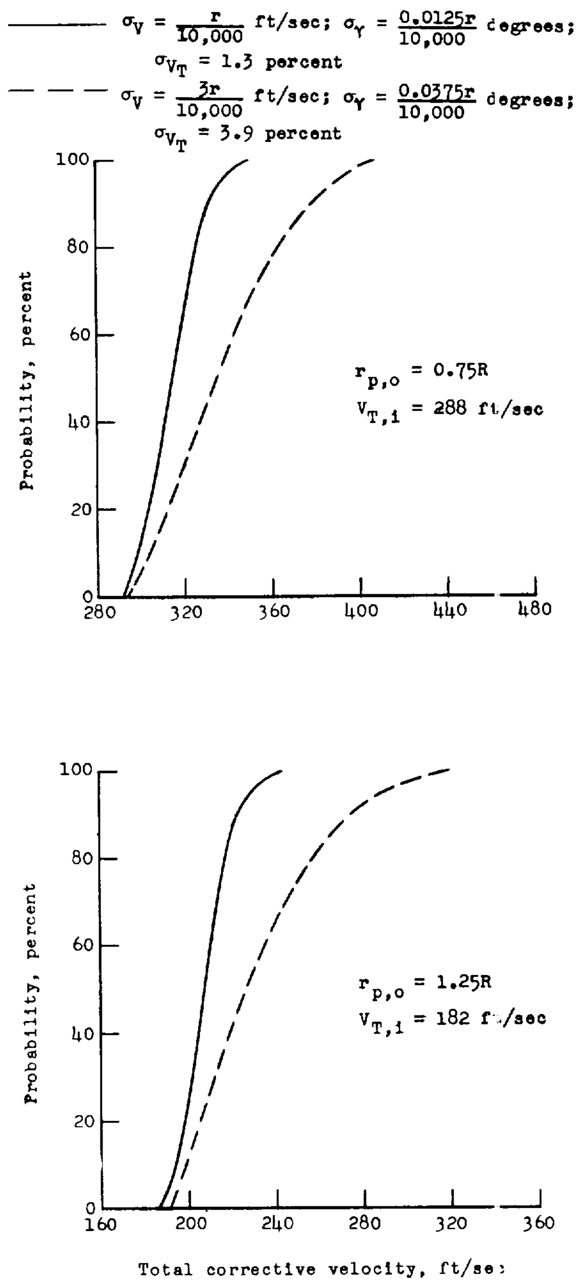


(b) σ deadband.

Figure 12.- Concluded.



(a) Zero deadband.

Figure 13.- Total-corrective-velocity probability-distribution curves for σ and zero deadbands using the angular method where corrections were made to the center of the deadband for two sets of instrumentation inaccuracies. $\theta_f = 10^\circ$.

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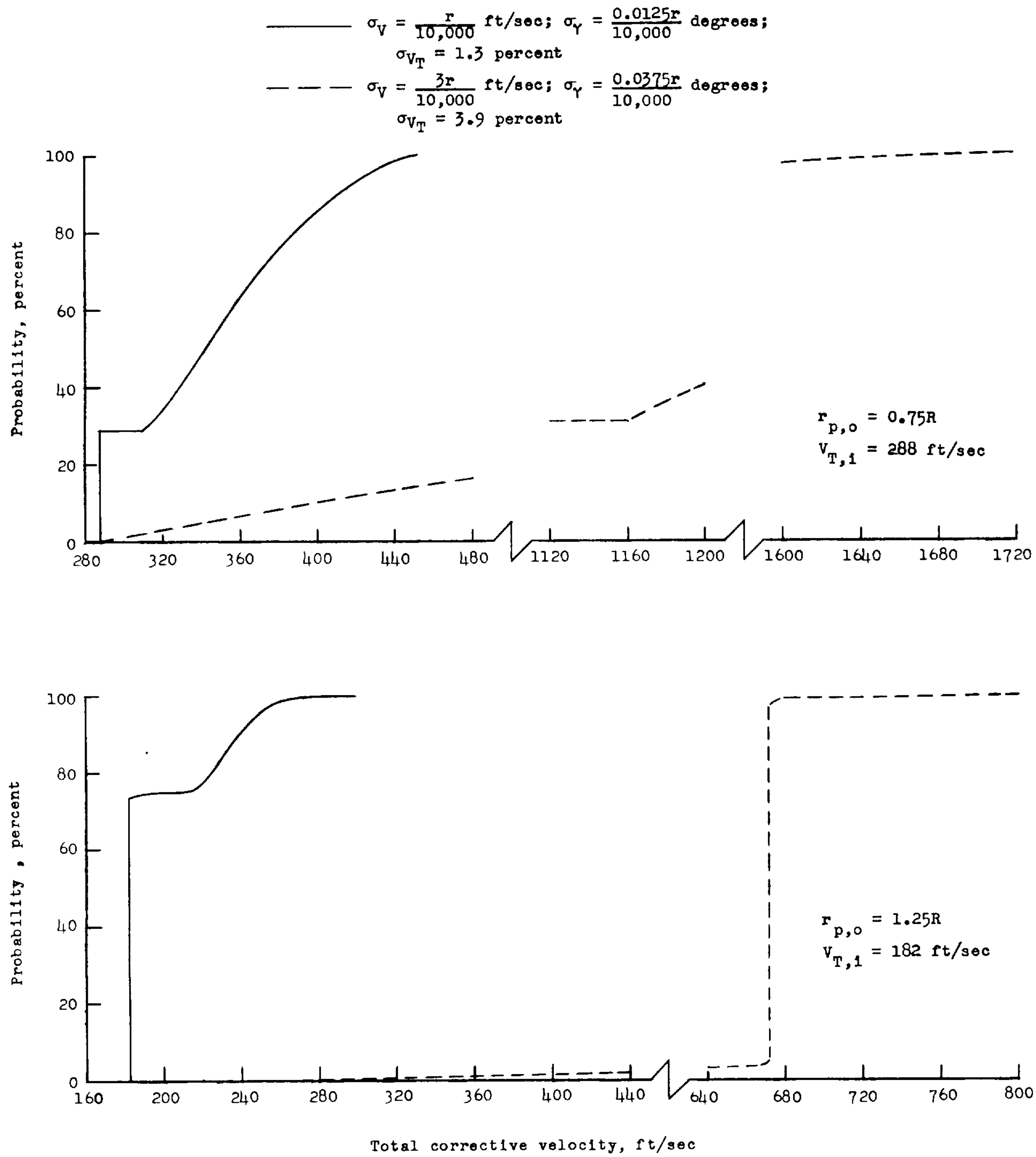

(b) σ deadband.

Figure 13.- Concluded.